EXERCISE – III SUBJECTIVE QUESTIONS

1. Line $\frac{x}{6}+\frac{y}{8}$ = 1 intersects the x and y axes at M and N respectively. If the coordinates of the point P lying inside the triangle OMN (where 'O' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find

(a) the coordinates of the point P and

(b) the radius of the circle escribed opposite to the angle N.

2. Two vertices of a triangle are (4, –3) & (–2, 5). If the orthocentre of the triangle is at (1, 2), find the coordinates of the third vertex.

3. The point A divides the join of P(–5, 1) & Q(3, 5) in the ratio K : 1. Find the two values of K for which the area of triangle ABC, where B is (1, 5) and C is (7,–2), is equal to 2 units in magnitude.

4. Determine the ratio in which the point P(3, 5) divides the join of A(1, 3) and B(7, 9). Find the harmonic conjugate of P w.r.t. A & B.

5. A line is such that its segment between the straight lines 5x – y – 4 = 0 and 3x + 4y – 4 = 0 is bisected at the point (1, 5). Obtain the equation.

6. A line through the point P(2, – 3) meets the lines x – 2y + 7 = 0 and x +3y – 3 = 0 at the points A and B respectively. If P divides AB externally in the ratio 3 : 2 then find the equation of the line AB.

7. The area of a triangle is 5. Two of its vertices are (2, 1) & (3, –2). The third vertex lies on y = x + 3. Find the third vertex.

8. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}$ = 1 & $\frac{x}{b}+\frac{y}{a}$ = 1, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve 2xy (a + b) = ab(x + y)

9. In the xy plane, the line 'ℓ_1' passes through the point (1, 1) and the line 'ℓ_2' passes through the point (–1, 1). If the difference of the slopes of the lines is 2. Find the locus of the point of intersection of the lines ℓ_1 and ℓ_2.

10. Two consecutive sides of a parallelogram are 4x + 5y = 0 & 7x + 2y = 0. If the equation to one diagonal is 11x + 7y = 9, find the equation to the other diagonal.

11. The line 3x + 2y = 24 meets the y-axis at A and the x-axis at B. The perpendicular bisector of AB meets the line through (0, –1) parallel to x-axis at C. Find the area of the triangle ABC.

12. If the straight line drawn through the point $P(\sqrt{3},2)$ and inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}$ x – 4y + 8 = 0 at Q. Find the length PQ.

13. Find the area of the triangle formed by the straight lines whose equations are x + 2y – 5 = 0, 2x + y – 7 = 0 and x – y + 1 = 0 without determining the coordinates of the vertices of the triangle. Also compute the tangent of the interior angles of the triangle and hence comment upon the nature of triangle.

14. A triangle has side lengths 18, 24 and 30. Find the area of the triangle whose vertices are the incentre, circumcentre and centroid of the triangle.

15. The points (1, 3) & (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line y = 2x + c. Find c and the remaining vertices.

16. A straight line L is perpendicular to the line 5x – y = 1. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

17. The triangle ABC, right angled at C, has median AD, BE and CF.AD lies along the line y = x + 3, BE lies along the line y = 2x + 4. If the length of the hypotenuse is 60, find the area of the triangle ABC.

18. Two equal sides of an isosceles triangle are given by the equations 7x – y + 3 = 0 and x + y – 3 = 0 and its third side passes through the point (1, – 10). Determine the equation of the third side.

19. The equations of the perpendicular bisectors of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$, respectively. If the point A is $(1, -2)$ find the equation of the line BC.

20. If $(x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2$
$(x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2$
and $(x_3 - x_1)^2 + (y_3 - y_1)^2 = c^2$

then $\lambda \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = (a + b + c)(b + c - a)(c + a - b)$

$(a + b - c)$. Find the value of λ.

21. Given vertices A(1, 1), B(4, –2) and C(5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

22. Triangle ABC lies in the Cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respectively and the coordinates of A are (p, q). The line containing the median to the side BC has slope –5. Find the largest possible value of $(p + q)$.

23. Determine the range of values of $\theta \in [0, 2\pi]$ for which the point $(\cos\theta, \sin\theta)$ lies inside the triangle formed by the lines $x + y = 2$; $x - y = 1$ and $6x + 2y - \sqrt{10} = 0$.

24. The points (–6, 1), (6, 10), (9, 6) and (–3, –3) are the vertices of a rectangle. If the area of the portion of this rectangle that lies above the x-axis is a/b, find the value of $(a + b)$, given a and b are coprime.

25. Let ABC be a triangle such that the coordinates of A are (–3, 1). Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$. Then match the entries of column-I with their corresponding correct entries of column-II.

Column–I	**Column–II**
(A) Equation of the line AB is	(P) $2x + y - 3 = 0$
(B) Equation of the line BC is	(Q) $2x - 3y + 9 = 0$
(C) Equation of the line CA is	(R) $4x + 7y + 5 = 0$
	(S) $18x - y - 49 = 0$